

*On the Vector Potential and the Representation of a Polyharmonic Function in n -Dimensions**

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Abstract. An n -dimensional finite region of flat space is assumed and the representation of a solenoidal vector field for such a region is studied. If, in addition, the region is assumed to possess a single boundary, it is shown that *every* (sufficiently smooth) solution of $\nabla^{2\beta}\varphi = 0$, for positive integer β , admits representation in terms of

$$\sum_{p=0}^{\beta-1} \binom{n+p-1}{p}$$

harmonic functions.

1. Introduction. In 1899, Almansi [2] first investigated the representation of a polyharmonic function in terms of a number of harmonic functions. He found that, in the case of three dimensions defined by rectangular Cartesian coordinates (\mathbf{x}) , solutions to the equation $\nabla^{2\beta}\varphi(\mathbf{x}) = 0$, where ∇^2 denotes the Laplacian operator, are given by β harmonic functions $\varphi^{(i)}(\mathbf{x})$ in the form

$$(1.1) \quad \varphi(\mathbf{x}) = \sum_{i=0}^{\beta-1} p^{(i)} \varphi^{(\beta-i)}.$$

Here, $p^{(i)}$ denotes a product of i linear functions of the coordinates (\mathbf{x}) , with $p^{(0)} = 1$, and β is a positive integer. Almansi's proof, of completeness, however, applies only to regions which possess certain directional convexities. (A region of space is said to be *convex* in a given direction if an extended line drawn in this direction intersects the boundary of the given region in at most two points.) Thus, it cannot be expected that *all* polyharmonic functions will admit his representation (1.1) without regard to the topology of the region. Later, Frank and von Mises [3, p. 848], guided by the work of Almansi [2], obtained two representations for the solution of the biharmonic equation in two dimensions

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