

Some Mixed Boundary Value Problems for Regions with Spherical Boundaries

R. P. SRIVASTAV & PREM NARAIN

Communicated by I. N. SNEDDON

1. Introduction. There has been considerable interest in the solution of boundary-value-problems with mixed type of conditions on the boundary. The problems which have received most attention however are those for which all the boundaries but one are so situated that the boundary-conditions on these can be ignored and the boundary-conditions are therefore replaced by the regularity conditions at infinity. Recently Collins [1] and Sneddon [2] have discussed the problem of an electrified disk inside an earthed hollow cylinder, the radius of the disk being assumed comparable with that of the cylinder. Collins [3], Sneddon and Tait [4] and Sneddon and Welch [5] using similar techniques have also examined the stress distribution inside an isotropic homogeneous elastic solid cylinder containing a penny-shaped crack. A different approach for the solution of such problems, based on the theory of dual series equations, has been given by Sneddon and Srivastav [6] and Srivastav [7], [8], [9].

In this paper we investigate the solution of boundary value problems for Laplace's equation where the mixed type of conditions are specified on a diametral plane of a sphere with a simple condition on a spherical boundary. Two problems are discussed; (a) in §2 the problem of an electrified disk inside an earthed sphere, and (b) in §3 the problem for a half-space with a hemispherical cavity. We formulate the problem in terms of integral equations and show that these integral equations can be reduced to a single Fredholm equation of the second kind. The latter in general is best solved numerically but can also be solved approximately, in some cases, by analytical methods. We conclude the paper by discussing in §4 an application of the theory developed to a torsion problem of infinitesimal elasticity.

The technique used in this paper for the formulation of the problem is very similar to that of Sneddon [2]. For the solution we rely heavily on the idea, introduced by Sneddon and Srivastav [6] of assuming a convenient integral representation for the sum-function in the region where its value is not specified. The solution given here differs in one respect from the usual solutions: *it is not a variable separable solution.*