

Anti-Locality of Certain Lorentz-Invariant Operators

I. E. SEGAL¹ & R. W. GOODMAN²

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1. Introduction. The non-relativistic approximation to a relativistic theory involves basically the estimate

$$(m^2 + v^2)^{1/2} = m + \frac{v^2}{2m} + O((v^2/m)^2)$$

valid for velocities v which are small relative to the velocity of light which is here taken as the unit (and hence small also relative to the finite mass m). The non-relativistic energy thus appears as the approximation given by the first two terms of the power series expansion. In quantum theory, the corresponding estimate involves operators: v^2 is replaced by $-\Delta$ (taking Planck's constant also as a unit), and the estimate becomes

$$(m^2 I - \Delta)^{(1/2)} = mI - \frac{\Delta}{2m} + O((\Delta/m)^2),$$

where I is the unit operator and Δ the Laplacian. This is easily made into a rigorous mathematical statement: if f is a square-integrable function whose Fourier transform \hat{f} has support in the region $[\mathbf{k} : |\mathbf{k}| < am]$, $a < 1$, (physically this means f represents a state in which only energies strictly less than the rest-energy m are involved), then $(m^2 I - \Delta)^{(1/2)} f$ is equal to the expression obtained from the series expansion of $(m^2 I - \Delta)^{(1/2)} f$, in powers of Δ , *etc.*

The partial sums of this power series expansion are all local operators, and in fact simply partial differential operators with constant coefficients. With or without an external potential V there is an extensive theory for such operators, especially for the operator $\Delta - V$ representing the non-trivial part of the linear approximation. The corresponding relativistic operators are more troublesome to deal with in general, so that it is a matter of interest to determine how far the qualitative features of the non-relativistic theory may be extrapolated to

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