

# *Compactification of Homogeneous Spaces, I*

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**1. Introduction.** The aim of this paper and those to follow is a development of ideas and techniques, mainly qualitative in nature, that were originally invented to treat the Poisson integral formula [4, 5] in non-compact Riemannian symmetric spaces, but that have—as we aim to show—a much wider validity. For example, there are relations to the current active application of Lie group ideas to mathematical physics. An announcement in [6] of the ideas can serve as a detailed introduction to this series. A main theorem to be proved in this paper is the following:

*A compact manifold of negative Euler characteristic can have no Lie group acting transitively on it.*

In addition, we will give a treatment of the theory of boundary components (in the sense of Piatetzki-Shapiro) for an arbitrary equivariant compactification of a symmetric space. Recall, for example, that Calvin Moore has given [8] a boundary component theory for the Furstenberg compactifications. The more geometric method to be given here shows some promise of extending to non-symmetric situations.

In general, we follow Helgason's notations [3]. All data will be  $C^\infty$  unless mentioned otherwise. If  $M$  is a manifold,  $F(M)$  denotes its ring of real-valued functions;  $V(M)$  its vector fields, considered both as an  $F(M)$ -module and as a real Lie algebra under the Jacobi bracket operation  $(X, Y) \rightarrow [X, Y]$ ;  $T(M)$  denotes its tangent bundle, with  $M_p$ , for  $p \in M$ , the tangent space to  $M$  at  $p$ . When talking about a submanifold of  $M$ , say  $N$ , we will, for notational convenience, think of  $N$  as identified with its image in  $M$ , and, for  $p \in N$ , identify  $N_p$  with its image in  $M_p$ .

I would like to thank Adam Koranyi and Calvin Moore for much invaluable help. I would also like to dedicate this work to the memory of David Lowdenslager: He was one of the first to recognize and push the importance of qualitative, geometric techniques in this circle of problems concerning analysis on symmetric spaces, and very generously taught me the basic ideas.

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