

On Filtered Lie Algebras and Geometric Structures III

SHOSHICHI KOBAYASHI AND
TADASHI NAGANO*

Communicated by S. S. CHERN

1. Introduction. In this paper we classify all irreducible linear Lie algebras of infinite type over an algebraically closed field of characteristic 0. In [3], E. Cartan gave the classification based on the case-by-case checking without giving its details; on p. 150 of [3], he states “Nous n’entrerons pas dans le détail des calculs qui conduisent à cette détermination, ces calculs étant assez longs et fastidieux”. In spite of a recent interest in the subject, no satisfactory proof of Cartan’s results has been found to date. We present here a complete and systematic proof avoiding the case-by-case verification, thus paving the way for the classification of primitive filtered Lie algebras, *i.e.*, infinite primitive pseudo-groups of transformations.

To explain our results and also to motivate definitions in §2, we introduce a filtration in a Lie algebra of vector fields as follows. Let M be a complex manifold of dimension n and L a Lie algebra of holomorphic vector fields on M . Fixing a point p of M , let \mathfrak{F} be the algebra of germs of holomorphic functions at p , *i.e.*, the algebra of converging power series at p . Let \mathfrak{g} be the maximal ideal of \mathfrak{F} , *i.e.*, the ideal consisting of holomorphic functions which vanish at p . Then, for each non-negative integer k , we set

$$L_k = \{X \in L; X(\mathfrak{g}) \subset \mathfrak{g}^{k+1}\}.$$

We also set $L_{-1} = L$. Thus, we have a decreasing sequence of subalgebras:

$$L = L_{-1} \supset L_0 \supset L_1 \supset L_2 \supset \cdots$$

We say that L is transitive if $\dim L/L_0 = \dim M$. Axiomatizing important properties possessed by a transitive $L = \{L_k\}$, we define a filtered Lie algebra abstractly. With each filtered Lie algebra $L = \{L_k\}$, we associate a graded Lie algebra $\sum_{k=-1}^{\infty} L_k/L_{k+1}$.

* Both authors are partially supported by NSF Grant GP-812.