

# *On the Non-solvability of Dirichlet's Problem for the Minimal Surface Equation\**

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1. Given a Jordan domain  $P$  in the  $(x, y)$ -plane and a continuous function  $f(x, y)$ , defined on its boundary  $\partial P$ , characterizing a simple closed curve  $\Gamma$  in  $(x, y, z)$ -space. The non-parametric Plateau problem, *i.e.* the Dirichlet problem for the minimal surface equation, consists in the determination of a solution  $z(x, y) \in C^2(P) \cap C^0(\bar{P})$  of this equation in  $P$  whose values on  $\partial P$  coincide with those of the function  $f$ .

In 1832 S. D. Poisson announced that he had obtained an existence proof for the case where the curve  $\Gamma$  differs only slightly from the  $(x, y)$ -plane. He never published this proof, however. In 1909 such a proof was put forward in a classical paper by A. Korn. Korn assumes the boundary  $\partial P$  as well as the function  $f$  to belong to class  $C^{2,\lambda}$  and requires that the  $(2, \lambda)$ -norm of  $f$  be sufficiently small. In later years the problem was dealt with in increasing generality in the work of S. Bernstein, A. Haar, Ch. Müntz, J. Schauder, and especially T. Radó, and it is now a well established fact that a solution exists for arbitrary continuous boundary data, *provided* the domain  $P$  is convex (not necessarily strictly convex).

2. For general domains the situation is more complex. This was first pointed out by S. Bernstein (see [1], esp. pp. 455–469) and discussed in greater generality by J. Leray [9]. Bernstein outlined a proof that for sufficiently non-linear elliptic equations (equations of order or “genre” 2—the minimal surface equation being a prototype of such equations) there are non-convex domains for which Dirichlet's problem, in general, is not solvable. The domains  $P$  he considered are bounded in part by a concave analytic arc  $BC'D$ , as in figure 1, and his proof is based on the construction of an auxiliary function  $z = \phi(x, y)$ , a solution of class  $C^2(P) \cap C^0(\bar{P})$ , whose gradient tends to infinity upon approach of any point of the arc  $BC'D$ . The auxiliary function  $\phi(x, y)$  is obtained as solution of a Cauchy problem in sufficiently small domains.

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