

# *Gaussian Processes and Generalized Schroedinger Equations\**

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**I. Introduction.** In [2] and [3], R. H. Cameron has defined several special integrals connected with the Wiener integral, and has studied the existence and properties of those integrals. Under certain hypotheses on the integrand, these integrals are connected with R. P. Feynman's function space integral [12], in the one dimensional case. These "Feynman" integrals of certain classes of functionals serve as solutions to the Schroedinger equation.

This paper extends these ideas from the Wiener stochastic process to Gaussian Markov stochastic processes. A further problem considered is an existence theorem for a partial differential equation which plays the role for Gaussian Markov processes that the generalized heat flow equation [4] does for the Wiener process.

A Gaussian Markov stochastic process  $\{x(t), a \leq t \leq b\}$  is determined by a mean function  $E\{x(t)\} = m(t)$  and a covariance function

$$r(s, t) = E\{[x(s) - m(s)][x(t) - m(t)]\}.$$

The Markovian nature dictates that  $r(s, t)$  has the "factorable" form:

$$r(s, t) = \begin{cases} u(s)v(t), & s \leq t \\ u(t)v(s), & t \leq s, \end{cases}$$

where  $u(t) \geq 0$ ,  $v(t) > 0$ , and  $u(t)/v(t)$  is strictly increasing on  $[a, b]$ . Furthermore we will require that  $u''(t)$  and  $v''(t)$  exist and are continuous on  $[a, b]$  so that almost all sample functions are continuous. [9]. Finally we require that

$$[v(t)u'(t) - u(t)v'(t)] > 0, a \leq t \leq b.$$

Five examples of such processes appear in Section VII.

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