

Dissipative Ordinary Differential Operators¹

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Introduction. We shall study the initial value problem for dissipative differential systems in one spatial variable of the form

$$(1.1) \quad y_t = (Ay)_x + By, \quad -\infty \leq a < x < b \leq \infty, \quad t > 0,$$

and

$$(1.2) \quad y_t = (Ay)_x + (By)_x + Cy, \quad -\infty \leq a < x < b \leq \infty, \quad t > 0$$

by means of the theory of semi-groups of operators.

Section 1 is devoted to the system (1.1). Here $y = (\eta^1, \eta^2, \dots, \eta^k)$ is a function of x and t with values in a k -dimensional complex Euclidean space E^k , A and B are $k \times k$ matrix-valued functions of x alone, A being Hermitian and of constant rank with p positive and n negative eigenvalues, which together with B satisfies the condition

$$D = B + B^* + A_x \leq 0, \quad a < x < b,$$

where B^* is the adjoint matrix to B under the usual inner product in E^k defined by

$$(y, z) = \sum_{i=1}^k \eta^i \bar{\zeta}^i, \quad \text{with} \quad |y| = [(y, y)]^{1/2}.$$

Further, we assume that the elements of $A(x)$ are absolutely continuous on each compact subinterval of (a, b) and that the elements of A_x and B are square integrable on each compact subinterval of (a, b) .

Physical considerations suggest that we impose boundary conditions on y such that

$$(1.3) \quad (Ay, y)^b - (Ay, y)^a \leq 0.$$

Boundary conditions of this kind will be called *dissipative*.

The Cauchy problem for the system (1.1) can now be formulated as follows: We require of the operator

$$(1.4) \quad Ly = (Ay)_x + By,$$

with a domain $D(L)$ suitably restricted by dissipative boundary conditions, that it generate a strongly continuous semi-group of linear bounded operators,

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