

# *Boundary Properties of Functions of Several Complex Variables<sup>1</sup>*

WALTER RUDIN AND E. L. STOUT

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**Introduction.** Every continuous complex function on the closed unit disc which is holomorphic in the open disc and which has constant absolute value on the unit circle is (as is well known) a finite Blaschke product. In particular, it is rational. The question arises whether anything like this is true for functions of several variables. The answer turns out to be affirmative; as a very special case of one of our main results (Theorem 2.1) we obtain the following: *If  $f$  is holomorphic in  $U^n$  (the open unit polycylinder, i.e., the cartesian product of  $n$  open unit discs  $U$ ) and continuous on its closure  $\bar{U}^n$ , and if  $|f| = 1$  on  $T^n$  (the distinguished boundary of  $U^n$ , the  $n$ -dimensional unit torus), then*

$$f(z_1, \dots, z_n) = \frac{M(z_1, \dots, z_n)P^*(1/z_1, \dots, 1/z_n)}{P(z_1, \dots, z_n)},$$

where  $M$  is a monomial,  $P$  is a polynomial without zeros in  $U^n$ , and  $P^*$  is the polynomial obtained from  $P$  by replacing the coefficients of  $P$  by their complex conjugates.

This extends the result of [4] and [6] where attention was restricted to entire functions. For a different generalization, see [11].

Two notions turn out to be important for the generalization of our above-quoted result. The first is that of a *determining set* or, briefly, *D-set*: If  $\mathfrak{F}$  is a class of functions on a set  $X$ , we call a set  $E \subset X$  a *D-set* for  $\mathfrak{F}$  if the assumptions  $f \in \mathfrak{F}$ ,  $g \in \mathfrak{F}$ ,  $f|E = g|E$  imply  $f = g$ . (As usual,  $f|E$  denotes the restriction of  $f$  to  $E$ .) In Section I we obtain some theorems about *D-sets* for the classes  $H(\Omega, E)$ : these consist of the functions which are continuous on  $\Omega \cup E$ , where  $E$  lies in the boundary of the open set  $\Omega$  in the complex  $n$ -space  $C^n$ , and which are holomorphic in  $\Omega$ .

The second concept we need is that of a *circular set*: If  $a = (a_1, \dots, a_n) \in Z^n$ , the group of lattice points in euclidean space  $R^n$  (i.e., if  $a_1, \dots, a_n$  are integers)

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