

Complex Homogeneous Contact Manifolds and Quaternionic Symmetric Spaces

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1. Introduction. The compact simply connected complex homogeneous contact manifolds M were studied by W. Boothby [1, 2]. He showed that the manifolds M are in one-one correspondence with the compact centerless Lie groups G_u , the correspondence being given by $M = G_u/L$ where L is a certain subgroup unique up to conjugacy. L is a local direct product $L_1 \cdot T$ where T is a circle group which defines the complex structure.

The compact simply connected quaternionic symmetric spaces S can be found in É. Cartan's list [4] of all irreducible Riemannian symmetric spaces by use of our Theorem 3.7. An examination of the list shows that the spaces S are in one-one correspondence with the compact centerless Lie groups G_u , the correspondence being given by $S = G_u/K$ where K is a certain subgroup unique up to conjugacy. K is a local direct product $K_1 \cdot A_1$ where $A_1 \cong Sp(1)$, multiplicative group of unit quaternions.

A further look shows that K_1 and L_1 are locally isomorphic. This suggests the possibility that $K_1 = L_1$, that $T \subset A_1$, and that there is a fibering $M \rightarrow S$ given by $G_u/L \rightarrow G_u/K$ with fibre $K/L = A_1/T$ isometric to the Riemann sphere. In this paper we will give *a priori* proofs of the suggested relations between manifolds M and spaces S . We also extend these relations to the noncompact case. As a preliminary, we develop the theory of quaternionic structures on Riemannian manifolds.

2. Preliminaries on complex contact structure. Let M be a complex manifold of odd complex dimension, $\dim_{\mathbb{C}} M = 2n + 1$. (All manifolds will be assumed Hausdorff, separable and connected.) A *complex contact structure* on M is a family $\{(U_i, \omega_i)\}$ where

- (i) $\{U_i\}$ is an open covering of M ,
- (ii) ω_i is a holomorphic 1-form on U_i such that $\omega_i \wedge (d\omega_i)^n \neq 0$ at every point of U_i ,

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