

Wave Propagation and Electromagnetic Shock Wave Formation in Transmission Lines

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Communicated by I. N. SNEDDON

1. Introduction. The wave propagation phenomena that occur in a linear distributed parameter transmission line have been extensively studied and are well understood [1, 2]. However, the method of analysis that is used for linear systems does not readily generalise to allow the study of nonlinear distributed parameter transmission lines. The reason for this is connected with the fact that it is customary to convert the two linear first order transmission line equations to a single second order equation, the equation of telegraphy, and then to apply the principle of superposibility of solutions to this equation in order to deduce the solution to the problem in question. However, when the line is nonlinear, the superposibility of solutions no longer applies, the voltage and current are no longer solutions to the simple linear equation of telegraphy and attempts to develop this type of approach further usually require that approximations be made to simplify the nonlinearity involved [3].

The formal equivalence that exists between the nonlinear transmission line equations and the one-dimensional isentropic gas flow equations has been noted and exploited by Landauer [4] and Riley [5] in order to study nonlinear transmission line behaviour. The basic properties of electromagnetic shock waves were established by Landauer who made a direct application of the methods of gas dynamics described by Courant and Friedrichs [6]. Riley applied Landauer's work to a nonlinear transmission line involving a voltage dependent capacitance and, also following Courant and Friedrichs, derived a simple numerical scheme for the study of the wave propagation before the formation of the electromagnetic shock wave. Neither author considered the problem of when a continuous wave propagating down a nonlinear transmission line first becomes discontinuous and forms an electromagnetic shock wave. This corresponds to the interesting problem of the development of jump discontinuities in solutions to nonlinear hyperbolic partial differential equations with Lipschitz continuous initial data which has been discussed elsewhere [7, 8, 9].

The primary task of this paper is to use the results of references [7, 8] in order to determine the time t_c and the distance x_c at which a continuous wave