

# *The Ridge of a Jordan Domain and Completely Plastic Torsion*

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**1. Introduction.** Consider a cylindrical bar twisted by terminal couples. For a small angle of twist per unit length, it was formulated by Saint Venant as a Neumann problem by using his semi-inverse method [1]. In a sense, St. Venant's solution to his problem is complete. Also, his conjecture for torsional rigidity has been completely confirmed by Pólya [2]. For pure torsions beyond linearly elastic range, some questions remain open although much progress has been made during the last three decades [4-8]. The object of the present communication is to present comprehensive results for completely plastic torsion of a cylindrical bar with simply connected cross sections.

According to the theory for elastic-plastic solids [4, 6, 7], the twisted bar will remain elastic if the applied torque is small. As the applied torque is increased to a certain critical point, some portion of the cylindrical surface of the bar will become plastic. Furthermore, the plastic portion shall continue to grow as the applied torque increases. It has been recognized that for completely plastic torsion, *i.e.*, the entire bar becomes plastic, it requires an infinitely large angle of twist. Accordingly, all large torsions will be of the elastic-plastic type. However, fully plastic torsion is the limiting case of a sequence of elastic-plastic torsions and hence the torque for full plastic torsion is the least upper bound of those for all large torsions. Also, a thorough understanding of full plastic torsion is helpful for treating the difficult elastic-plastic torsion problem. In fact, to formulate the elastic-plastic torsion it is necessary to guarantee the existence of solution of fully plastic torsion so as to be able to prescribe the boundary conditions on the unknown boundary which separates the elastic and the plastic regions [3, 6].

Since discussions about the physical aspect of plastic torsion can be found in the literature [6, 7], we shall simply state it as a mathematically well-posed problem in the sense that its solution exists and is unique. Apparently, a precise statement for this problem appears for the first time. Also, it is not so obvious that such an uncommon boundary-value should be a well-posed one. It turns out that this problem is essentially a geometrical problem in the large; and the