

# *Evolution Equations Non-Linear in the Time Derivative*

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1. Considered primarily in this paper are evolution equations of the form

$$(1.1) \quad \beta(t) \frac{du}{dt} + A(t)u(t) = f(t), \quad 0 \leq t \leq T,$$

where  $T$  is arbitrary, the unknown  $u$  and given  $f$  are functions of  $t$  with values in appropriate Banach spaces, each  $A(t)$  is a linear "elliptic" operator and each  $\beta(t)$  is a non-linear operator mapping a Banach space into its dual. The basic question is whether (1.1) together with the initial condition  $u(0) = u_0$  is a well-posed problem.

An important example is afforded by the equation

$$(1.2) \quad \varphi(\partial u / \partial t) - \Delta_x u = 0,$$

in the cylinder  $(0, T) \times \Omega$ ,  $\Omega \subset E^n$ , ( $u = u(t, x)$  real-valued), where  $\varphi$  is an increasing, continuous function of a real variable. It is formally equivalent to the *non-linear heat equation*

$$(1.3) \quad c\rho \frac{\partial v}{\partial t} = \operatorname{div}_x (\kappa \operatorname{grad}_x v),$$

( $v = v(t, x)$ ), where the positive quantities  $c$  (specific heat),  $\rho$  (density) and  $\kappa$  (heat conductivity) each depend on the temperature  $v$ . Indeed, let  $k$  be an indefinite integral of  $\kappa$  and let  $\varphi$  be a function satisfying  $\varphi' \circ k = c\rho/\kappa$ . Then the solutions of (1.2) and (1.3) are related by the equation  $\partial u / \partial t = k \circ v$  provided the initial data are related by  $\varphi(k(v(0, x))) = \Delta_x u(0, x)$ .

In [8], using a finite-difference scheme in  $x$  and  $t$  and a compactness method, Kamenomostskaya specifically studied equation (1.2) (more precisely, the equation satisfied by  $\partial u / \partial t$ , which is equivalent). In the present paper we have in mind more general equations such as

$$(1.4) \quad \sum_i D_i^* \varphi_i(D_i(\partial u / \partial t)) - \Delta_x u = f(t, x),$$

in a cylinder  $(0, T) \times \Omega$  subject to an initial condition and various types of boundary conditions, where each  $D_i$  is a linear differential operator with formal

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