

# *Distributional Boundary Values of Functions Holomorphic in a Half Plane*

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**I. Introduction.** A classical and well known theorem states that complex valued functions  $F(\omega) \in L_2(-\infty, \infty)$  are boundary values in the  $L_2$  norm of functions

$$F(p) \equiv F(\sigma + i\omega) \in H^2, \quad \text{as } \sigma \rightarrow 0,$$

if and only if  $F(\omega)$  is the Fourier transform of some  $f(t) \in L_2(0, \infty)$ . Here,  $H^2$  denotes holomorphic functions  $F(\sigma + i\omega)$  which are  $L_2$  in the right half plane as a function of  $\omega$ , uniformly for all  $\sigma > 0$ . Moreover, according to Titchmarsh [1],  $F(p)$  is the Laplace transform of  $f$  (the one sided Paley-Weiner theorem) and the real and imaginary components of  $f$  satisfy reciprocal Hilbert transforms. The importance of this theorem and similar results on the boundary behavior of holomorphic functions in a half plane is, of course, well recognized (*c.f.* Hoffman [5]). Recently, however, we were led to extend the Titchmarsh theorem to consider boundary values of a class of functions which are holomorphic and of algebraic growth at infinity. The boundary values belong to a distributional class which contains  $L_2$ . More precisely, it was possible to give necessary and sufficient conditions so that the class  $H^+$  of functions, which are holomorphic in the half plane  $\operatorname{Re} p > 0$  and bounded by a polynomial uniformly in every half plane  $\operatorname{Re} p \geq \sigma > 0$ , should converge to a distribution in  $S'$  (tempered distribution). The condition is that the distribution in  $S'$  should be the Fourier transform of something in  $S' \cap D'_+$  (tempered distributions having their support in the positive half axis) whose Laplace transform is, in turn, the function in  $H^+$ . This theorem was proven in [2] as a variant of a previous result of H. A. Lauwerier [3], where it is shown that every function in  $H^+$  converges to the Gelfand transform of some distribution in  $D'_+$ . The theorem of Lauwerier does not, however, yield information on the tempered boundary behavior of the half plane function. It will be shown herein that not every function in  $H^+$  has  $S'$  boundary values (convergence is in the  $S'$  topology), and that the conditions for our theorem to hold are the best possible.