

On Filtered Lie Algebras and Geometric Structures, IV

SHOSHICHI KOBAYASHI* &
TADASHI NAGANO**

Communicated by S. S. CHERN

1. Introduction. In the present paper, we shall give some conditions for a filtered Lie algebra to be isomorphic with its associated graded Lie algebra. We recall [4] that a *filtered Lie algebra* L (over a field \mathbf{F} of characteristic 0) is a Lie algebra with a decreasing sequence of subalgebras

$$L = L_{-1} \supset L_0 \supset L_1 \supset L_2 \supset \cdots,$$

such that

- (a) $\bigcap_p L_p = 0$;
- (b) $[L_p, L_q] \subset L_{p+q}$;
- (c) $\dim L_p/L_{p+1} < \infty$;
- (d) For every $t \in L_p$, $p \geq 0$, such that $t \notin L_{p+1}$, there is an element $x \in L$ such that $[t, x] \notin L_p$.

Given a filtered Lie algebra $L = \{L_p\}$, its *associated graded Lie algebra* $\mathcal{G}(L)$ is defined as follows:

$$\mathcal{G}(L) = \sum_{p=-1}^{\infty} G_p, \quad G_p = L_p/L_{p+1},$$

the bracket operation being induced in a natural way. Corresponding to conditions (b), (c) and (d), the associated graded Lie algebra satisfies the following conditions (there is no condition corresponding to (a)):

- (b') $[G_p, G_q] \subset G_{p+q}$;
- (c') $\dim G_p < \infty$;
- (d') For every nonzero $t \in G_p$, $p \geq 0$, there is an element $x \in G_{-1}$ such that $[t, x] \neq 0$.

* Sloan Fellow, partially supported also by NSF Grant GP-812.

** Partially supported by NSF Grant GP-812.