

Generalized Variational Principles for Convective Heat Transfer and Irreversible Thermodynamics

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Abstract. Variational principles for convective heat transfer are extended to nonhomogeneous fluids and to fluids with thermal expansion. The results apply to nonlinear problems with temperature dependent properties. The context is restricted to heat transfer but the derivation and a discussion of the physics brings out the general validity of the variational principles for a large category of irreversible processes.

1. Introduction. Variational methods in heat transfer for both linear and non-linear problems were obtained as an extension of general thermodynamic principles [1] [2]. The method is applicable to convective heat transfer in a fluid. For the fluid with constant properties a variational principle was derived from the classical linear equations for the temperature field [3]. It was shown that an approach based on thermodynamic concepts provides a very general form of the variational principle which point to its validity in a large class of dissipative phenomena and is applicable to the non-linear problem of convective heat transfer in a fluid with temperature-dependent properties [4].

The aforementioned results were derived for an incompressible homogeneous fluid. We will extend the variational method to fluids composed of heterogeneous particles and to a fluid with thermal expansion. This generalization introduces the concept of thermal potential of an individual fluid particle. The derivation is obtained in a concise form which brings out the mathematical structure and the physical significance of the results. In particular we have discussed the analogy with the treatment of holonomic systems in classical mechanics, the potential development of the method in general thermodynamics as well as applications to turbulent flow, internal heat generation and mixed fluid-solid systems such as porous media.

The derivation is based on a simple but unconventional formulation. Its equivalence to the more familiar equation for convective phenomena is clarified in the last paragraph.