

# *Uniqueness of the Axisymmetric Exterior Traction Problem in Elastostatics*

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**1. Introduction.** This paper considers the uniqueness of the axisymmetric exterior traction boundary value problem in the linear theory of elastostatics. The treatment depends upon a transformation due to Aleksandrov [1] which converts the displacement and stress of a plane strain solution into those of a related axisymmetric solution. The axisymmetric stress generated in this fashion is shown to be complete and to possess a one-one correspondence to the plane stress. By means of these properties sufficient conditions are derived for the uniqueness of the axisymmetric problem from those already known in the plane strain problem. Necessary conditions are separately established by appealing to counter-examples.

We prove that uniqueness exists if and only if  $\nu \neq +1$ ,  $\mu \neq 0$ , where  $\nu$  and  $\mu$  are respectively Poisson's ratio and the shear modulus. The same conclusion is valid for the axisymmetric internal traction problem [2], but is a distinct improvement on the general three-dimensional exterior problem. Here, only the sufficient condition  $-1 < \nu < 1$ ,  $\mu \neq 0$ , is so far known [3] in the literature.

In [2], conditions for uniqueness in the internal problem have been established by a similar procedure to the one followed in this paper. The difference lies in the structure of the Aleksandrov transformations, since the one employed here permits a weaker rate of decay at infinity of the stress components.

For the axisymmetric exterior displacement problem the method of this paper produces no new information. The situation is identical to the general case, where it is known that uniqueness holds if  $-\infty < \nu \leq \frac{1}{2}$ ,  $1 < \nu < \infty$ ,  $\mu \neq 0$ . The proof has been given by Fichera [4], Duffin and Noll [5], Gurtin and Sternberg [6], and Finn and Chang [7], among others. We wish only to add that uniqueness also holds for  $\nu = \pm \infty$ ,  $\mu \neq 0$ , since for these values of the moduli the displacement becomes harmonic (see Section 5), and the conclusion therefore follows trivially from potential theory.

**2. The Aleksandrov transformation.** Consider a state of plane strain in a long cylindrical bar whose generators are parallel to the  $x$ -axis. Let the cross-