

*A Theorem on Spinor-Tensors**

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Communicated by V. HLAVATÝ

Introduction. Recently Lichnerowicz [4], as part of his general theory of propagators, has been considering spinors and spinor-tensors on a four dimensional Lorentzian manifold. The main ideas of his work can be carried over into n -dimensional Lorentzian manifolds. In these manifolds one can define objects which we call spinor-tensors of type $\begin{pmatrix} rs \\ pq \end{pmatrix}$, i.e., objects whose components have r contravariant spinor indices, s covariant spinor indices, p -contravariant tensor indices and q -covariant tensor indices.

In this first section we shall discuss the existence of spinor fields and will give the general definition of spinor tensors. Then we will define a Laplacian for spinor-tensors and prove that it possesses the familiar property $\langle \Delta\varphi, \psi \rangle = \langle \varphi, \Delta\psi \rangle$.

Throughout this paper the notation and conventions of Lichnerowicz [3] are used exclusively.

1. Spinor-tensors. Let V_n be an n -dimensional differentiable manifold. Consider the set of orthonormal frames on V_n which are the elements of a principal fibre bundle $\xi(V_n)$ which has the rotation group $SO(n)$ as its structural group. It is well known, Chevalley [1], that the spinor group $\text{Spin}(n)$ is a covering group of order two of $SO(n)$. Haefliger [2] has shown that $\xi(V_n)$ may be extended into another principal fibre bundle $\mathcal{S}(V_n)$, which has $\text{Spin}(n)$ as its structural group, if and only if the second Stiefel-Whitney characteristic class of V_n vanishes. When a manifold allows this extension we shall say that it has a spin structure and we restrict our discussion to differentiable manifolds which have this property. For example, this requirement excludes the complex projective plane. For questions of the uniqueness of spin structures see Milnor [5].

A point z of $\mathcal{S}(V_n)$ is a spinor frame and $\mathcal{S}(V_n)$ will be called the principal fibre bundle of spinor frames over V_n . Let π be a canonical projection of $\mathcal{S}(V_n)$ onto V_n and p be the projection of $\mathcal{S}(V_n)$ onto $\xi(V_n)$. The components of a

* This work was supported by the Air Force Office of Scientific Research Grant AFOSR-153-63.