

*Singular Symmetric Positive First Order Differential Operators**

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The study of first order symmetric positive differential operators was initiated by Friedrichs [2] in an effort to devise a theory which includes differential equations of mixed type. The differential systems considered in this theory are of the form

$$(1) \quad Ku = \sum_{\lambda=1}^r A^\lambda \frac{\partial u}{\partial x_\lambda} + Cu = f,$$

for x in a domain G contained in R^r , where u is a vector-valued function, the $\{A^\lambda(x)\}$ being symmetric matrix-valued functions continuously differentiable in \bar{G} , and C being a continuous matrix-valued function in \bar{G} . Such a system is called positive if

$$(2) \quad 2k \equiv C + C' - \sum \frac{\partial A^\lambda}{\partial x_\lambda} > 0, \quad x \in G.$$

In order that this problem be well-posed suitable boundary conditions must be imposed. The most convenient way to define the boundary conditions is in terms of the boundary matrix

$$(3) \quad B(x) = \sum A^\lambda(x)n_\lambda(x),$$

where $\{n_\lambda(x)\}$ are the components of the outer normal to the boundary β at x . At each point x of β a maximal positive (relative to $B(x)$) subspace $N_+(x)$ is chosen and the boundary condition is simply that $u(x)$ lie in $N_+(x)$ at each point of β . Friedrichs showed that this problem was well-posed provided that $B(x)$ was of constant rank near β , that β was of class C^2 , and that $N_+(x)$ contained no null vectors (relative to the non-singular part of $B(x)$) and was smoothly varying on β . Later Lax and Phillips [4] obtained a more direct proof which allowed for null vectors in $N_+(x)$. A somewhat different approach was developed by Sarason [10] who treated the inhomogeneous boundary value problem assuming again that $N_+(x)$ had no null vectors. However Sarason

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