

*Invariant Sublogics as a Way From Scalar to Many-Component Wave Equations**

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Our purpose is to derive the Pauli equation for the motion of a small charged body in an electromagnetic field from the Schroedinger equation for the same system. Now the former equation is of the form $P\psi = 0$, where ψ is a matrix-valued function on 3-dimensional euclidean space, while the latter is of the form $S\varphi = 0$, where φ is a complex valued function defined on a 6-dimensional configuration space. (Besides this they both contain the time parameter, of course.) The derivation proceeds by associating with (the one-dimensional space determined in its function space by) each ψ a 2-dimensional subspace $W(\psi)$ of the Hilbert space of φ 's, where $W(\psi)$ is invariant under the action of $SU(2)$ in that Hilbert space generated by its role as the universal covering group of the proper rotation group. Since the Schroedinger equation tells how these subspaces move with time, a motion of the ψ 's is defined, and this turns out to be just that given by the Pauli equation, provided that the mean square radius of the charge is exactly twice that of the mass for the body.

The idea of associating $SU(2)$ -invariant subspaces of the Hilbert space of ψ 's is based on an analysis of the states of the sublogic (to use the term of Birkhoff and v. Neumann) of those questions (Mackey's term) which are invariant under the action of a group (in this case $SU(2)$). This situation is explored in general, including the non-quantum case, but the details are worked out only for the quantum system of the charged body. It turns out that this invariant sublogic is the logic of a hyperselective system, whose pure states are consequently given by vectors of a Hilbert space (including the Pauli ψ 's) which represent invariant states of the various possible types of spin, and Pauli-like equations can be set up for each of the states of pure spin. Transition from one type to another is impossible. The types are characterized by the irreducible representations of $SU(2)$.

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