

*On Theorems of Hurwitz and Bochner**

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A classical theorem of Hurwitz [3] states that on a compact Riemann surface of genus >1 each holomorphic self-mapping that is homotopic to the identity reduces to the identity mapping. If one recalls that such a surface can be endowed with a Riemannian metric of curvature $K = -1$, and in this metric each holomorphic self-mapping becomes an isometry, one can then state the result in purely differential geometric terms. In this note we prove a generalization of this geometric version. Our generalization can be considered as an "extension" of a, by now, classical theorem of Bochner, the extension being from the infinitesimal isometries of Bochner's situation to isometries that are "small" in the sense of being homotopic to the identity. This extension is paid for by requiring, apparently, a stronger condition on the curvature of the space. While Bochner's result is not contained in our result, a small perturbation of our proof does yield his theorem, and we have included this proof in this note.

Our results and methods have many points in common with earlier work of Preissmann [6], Busemann [2], and with the very recent work of J. Wolf [8]. In particular, Wolf's results can be used to greatly shorten the proof of Theorem 1. A discussion of this possibility is given directly after the statement of our Theorem 2 and we respectfully advise the reader to glance at that discussion first in order to gain some perspective of the situation.

Finally, we should like to remark that our Theorem 2 gives easily a new proof of a theorem of Kobayashi [4].

The theorem given directly below is the generalization referred to above. We have stated it in its most simple form but the proof will show more, and just how much more will be indicated after the proof.

Theorem. *Let M_n be a compact Riemannian manifold with non-positive curvature $K \leq 0$ and with negative Ricci curvature. If $\Phi : M \rightarrow M$ is an isometry that is homotopic to the identity, then Φ is the identity map.*

Proof. Let M' be the universal covering manifold of M and $\pi : M' \rightarrow M$ the covering map. In the lifted metric π is a local isometry. Let $h : M \times I \rightarrow M$

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