

Sound Waves in Relativistic Magnetohydrodynamics

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1. Introduction. The mathematical treatment of the coupled motion of hydrodynamic flow and electromagnetic fields has assumed great importance recently. In the study of non-relativistic magnetohydrodynamics it is customary to adopt only the quasi-equilibrium approximation to the electrodynamic equations in which the displacement currents and charge accumulations are ignored. Such an analysis merely allows the possibility that the velocities, both random and ordered, of fluid elements are comparable with the classical velocity of sound. Recently various attempts have been made to give a covariant formulation to magnetohydrodynamics in which the above-mentioned approximations are not introduced. In this paper we are interested in the motion of sonic discontinuities with such a covariant formulation. Pioneering work in the field of wave motion has been done by de Hoffman and Teller [7], Reichel [11], Zumino [16], Akhieser and Polovin [1], Coburn [2, 3], Bruhat [5], Saini [12], Giere [6] and others. There are some disagreements in various results obtained by these authors. We plan to compare and extend the salient features of their analysis in the following sections.

Relativistic hydrodynamics has been formulated and put on firm foundations by various authors, while the equations of electrodynamics lend themselves readily into covariant description. Therefore the coupling of these two fields becomes rather simple. Recently Thomas [15] and Edlen [4] have given a discussion of relativistic hydrodynamics and re-defined the equations governing the flow of a perfect relativistic gas. We couple these modified equations with those of electrodynamics. The total energy tensor, thus obtained, enables us to compare various known results on wave motion. We consider only those fluids which have infinite conductivity.

We consider a weak disturbance whose propagation can be represented by a time-like hypersurface in an Einstein-Riemann space. By using the concept of singular surfaces across which all the magnetohydrodynamic quantities

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