

On Certain Reversed Processes and their Applications to Potential Theory and Boundary Theory

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Introduction. Let H_t be a transition function defined on a locally compact separable Hausdorff space. A (stationary) Markov process x_t with the basic probability space $(\Omega, \mathfrak{F}, P)$ and the terminal time ζ is said to be an H_t -process if H_t is the transition function of x_t and if almost all sample paths are continuous on the right and have limits from the left for all $0 < t < \zeta$. Given a non-negative random variable $\tau (\leq \zeta)$, we will define the reversed path \hat{x}_t by $\lim_{t \rightarrow 0} x_{\tau-t}$, if $0 < t < \tau < \infty$ and its terminal time $\hat{\zeta}$ by τ if $\tau < \infty$. The following two questions will be fundamental in the study of the reversed process $(\hat{x}_t, \hat{\zeta}, P)$:

1. When is the reversed process a (stationary) Markov process?
2. What is the transition function of the reversed process?

Nagasawa [9] discussed these problems in a general setting. He introduced the notion of a random time of type L and showed, in three different cases, that the above two problems are solved for the reversed process from a random time of type L .

In the present paper we will study reversed processes in a different context from that of [9]. This amounts to re-establishing or refining the results on potential and boundary theory in [7]. Incidentally the intrinsic relations between the three cases of [9] will be shown; Nagasawa disposed of those cases separately. For instance, the assumption B.3.2' in the second case of [9] is proved under certain hypotheses which are closely connected with those in the first case (see Theorem 4.2). Also compare Theorem 6.1 with the assumption C.3.1 of [9].

The main tool of our approach is the superharmonic transform (or h -path process in the terminology of Doob [1]) some basic material on which will be given in Sections 1, 2 and the Appendix. Let u be an excessive function relative

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