

# *On a Finite Lebedev Transform.*

## *Part 2.*

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In [2, 3] Kontorovich-Lebedev transforms were developed to deal with the operator  $(r^2\psi_{rr} + r\psi_r - k^2r^2\psi)$  on the intervals  $0 < r < a$ ,  $a < r < \infty$  where  $a > 0$ . Sturm-Liouville type boundary conditions or special cases thereof were imposed on  $r = a$ . This work is now supplemented by the construction of a similar transform adapted to the finite interval  $a < r < b$  where  $a > 0$ . The simplest example of such a transform is the following. In the usual notation for the modified Bessel functions we define the function

$$\psi_u(kr) = I_u(kr)K_u(kb) - I_u(kb)K_u(kr),$$

and introduce for a given function  $f(r)$  the transform

$$F(u_n) = \int_a^b \psi_{u_n}(kr)f(r) \frac{dr}{r},$$

where  $u_n$  is a zero of the function  $\psi_u(ka)$ . The inversion formula associated with this integral transform is most advantageously expressed as a contour integral in the complex  $u$ -plane. When the transform is applied to the solution of boundary value problems connected with partial differential equations the inversion formula leads directly to two different series expansions of the function  $f(r)$  on the stated interval. One such series is the Sturm-Liouville expansion in which the summation is taken over the above mentioned zeros. In the limiting case  $k \rightarrow 0$  the Lebedev transform degenerates into a Mellin type transform adapted to handle the same finite interval. In all cases the use of these transforms leads directly to both the classical series expansion and the Lebedev expansion without recourse to Watson's transformation.

1. Suppose that  $k$  is real and positive and consider the equation

$$(1) \quad r^2w_{rr} + rw_r - (k^2r^2 + u^2)w = 0.$$

This is Bessel's modified equation with basic solutions  $I_u(kr)$  and  $K_u(kr)$ . If we consider the finite interval  $0 < a \leq r \leq b$  with the condition of vanishing