

# *On the Axiomatic Foundations of the Theories of Directed Linear Graphs, Electrical Networks and Network-Programming\**

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**1. Introduction.** Matroid-theory, founded by Hassler Whitney [41] in 1935, is an abstract combinatorial theory with ramifications into algebra (theory of linear dependence [41], lattice theory [36]), projective geometry [39], electrical network theory [18], switching theory [13], and linear programming. In this paper, it is proposed to develop matroid-theory from the beginning in such a way as to put the phenomenon of *duality* into the forefront by starting with a self-dual axiom-system, following the model of projective geometry. The paper is, however, also intended as an expository paper on matroid-theory for the electrical engineer, the graph-theorist, and in fact the general mathematician. It is complete for each of these audiences, and the reader may skip over any remark obviously intended for another class of readers.

One of the main contributions of this theory to electrical theory is the clarification of the “duality principle,” which every electrical engineer knows, but few can state so exactly that it is *provable*. (Since this paper was written, the writer has become aware of the work of members of the Research Association for Applied Geometry (R.A.A.G.) on duality-theory. A representative of that group, M. Iri, has compared their point of view with the present one in the paper [18].) By the end of this paper, it will be so clear that we shall not even find it necessary to state it! Another purpose will be to help clarify the meaning of the contentions of G. Kron [21] that “graphs are illegitimate tools for studying network-theory” and that “1-networks have no nodes.”

The axiom-system we shall work with is highly appropriate for the treatment

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