

Quasi-nilpotent Equivalence of not Necessarily Commuting Operators.

I. COLOJOARĂ & C. FOIAȘ

Communicated by J. T. SCHWARTZ

Introduction. Let \mathfrak{X} be a Banach space, $\mathfrak{L}(\mathfrak{X})$ the Banach algebra of the linear, bounded operators of \mathfrak{X} , $\mathcal{P}_{\mathfrak{X}}$ the set of the projectors of \mathfrak{X} and \mathfrak{B} the family of the borelian sets of the complex plane C . A mapping $E : \mathfrak{B} \rightarrow \mathcal{P}_{\mathfrak{X}}$ is called a *spectral measure* if

- 1) $E(B_1 \cap B_2) = E(B_1)E(B_2), \quad (B_1, B_2 \in \mathfrak{B}),$
- 2) $E\left(\bigcup_{n=1}^{\infty} B_n\right)x = \sum_{n=1}^{\infty} E(B_n)x, \quad (B_n \in \mathfrak{B}, B_n \cap B_m = \emptyset \text{ if } n \neq m, x \in \mathfrak{X}),$
- 3) $E(C) = I,$
- 4) $\sup_{B \in \mathfrak{B}} \|E(B)\| < \infty.$

An operator $T \in \mathfrak{L}(\mathfrak{X})$ is called *spectral* if there exists a spectral measure E such that

- (i) $TE(B) = E(B)T, \quad (B \in \mathfrak{B}),$
- (ii) $\sigma(T | E(B)\mathfrak{X}) \subset \bar{B}, \quad (B \in \mathfrak{B}).$

The spectral measure E verifying (i) and (ii) is uniquely determined by T . It is called the spectral measure of T . A spectral operator S is said to be of *scalar type* (or simply *scalar*) if it has the form

$$S = \int_{\sigma} \lambda dE_{\lambda},$$

where E is its spectral measure.

These notions, of spectral operator and of scalar operator have been introduced by Dunford in [12], [13]. An operator $Q \in \mathfrak{L}(\mathfrak{X})$ is called quasi-nilpotent if $\lim_{n \rightarrow \infty} \|Q^n\|^{1/n} = 0$, or, equivalently if $\sigma(Q) = \{0\}$. Dunford has shown ([12], [13]) that an operator T is spectral if and only if it has the form $T = S + Q$, where S is a scalar operator and Q a quasi-nilpotent operator, which commutes