

# *The Development of Singularities of Solutions of Nonlinear Hyperbolic Equations of Order Greater than Unity*

A. JEFFREY

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**1. Introduction.** It is well known that initial value problems of nonlinear hyperbolic partial differential equations may develop singularities in their solutions after only a finite elapsed time. At these singularities the derivatives become unbounded and the solution of the equation ceases to exist in its elementary differentiable form. In this paper we make a simple application of earlier work [1] to determine the exact time of occurrence of special singularities of the solution of single nonlinear hyperbolic equations of order greater than unity involving two independent variables that are subject to Lipschitz continuous initial data.

That singularities in the solution of a nonlinear hyperbolic equation are not necessarily attributable to discontinuities in the initial data is well known and examples can easily be found [2] in which singularities still occur even when the prescribed initial data is analytic. Specifically, we shall examine the propagation of a Lipschitz discontinuity on the initial line and determine the exact time at which the derivatives of the solution first become unbounded and a jump discontinuity in the solution forms.

These problems have considerable physical interest since they are characteristic of certain types of wave propagation involving a *wavefront* which can be considered to form the boundary between two regions in which different solutions of the equation represent different physical states. The solution itself is continuous across such a wavefront but its derivatives normal to the wavefront may be discontinuous when crossing it, since the wave is usually assumed to be propagating into a *known* different state. Although in some problems no significance can be attached to any extension of a solution across a singularity there is a large class of problems in which the occurrence of a singularity simply indicates that a fundamental change has occurred in the mode of propagation of the disturbance. In physical applications singularities thus