

*On Similar Points of a Graph*¹

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Communicated by G.-C. ROTA

Much information in graph theory (as well, no doubt, as in other branches of mathematics) is lost when a seemingly reasonable conjecture turns out to be not quite always true and its counter-example is discarded. The purpose of the present paper is to preserve such a counterexample from oblivion and to answer several questions suggested by it. The problem which led to these results is itself becoming quite a well known conjecture in graph theory. Most of the graph theoretic terminology used in this paper may be found in [3]. We write $G \cong H$ to indicate isomorphism.

Ulam's Conjecture [7]. *If G and H are two graphs with p points v_i and u_i , respectively such that for all i , $G - v_i \cong H - u_i$, then $G \cong H$.*

Very briefly, the history of this conjecture is as follows:

Ulam formulated the problem and Kelly [5] solved it for trees and disconnected graphs. Harary [2] also did not succeed but studied the problem in terms of reconstructing a graph from the collection of its subgraphs obtained by removing one point at a time. Others have tried to prove the conjecture without success, although some convinced themselves that they had produced a valid proof. In one of these attempts, the following statement was used as one of the steps in a "proof."

Two points of a graph G are called *similar* if there is an automorphism of G sending one point onto the other. Of course, if u and v are similar points of G , then $G - u$ and $G - v$ are isomorphic graphs. The statement states the converse proposition.

Statement S. *If u and v are two points of a graph G such that the subgraphs $G - u$ and $G - v$ are isomorphic, then u and v are similar points.*

Statement S is not true: here are two counterexamples, G_1 and G_2 , each having eight points. In both of these graphs, $G_i - u_i \cong G_i - v_i$ (isomorphic), but u_i and v_i are not similar.

¹ Work supported in part by the U. S. Air Force Office of Scientific Research under grant AF-AFOSR-754-65.