

*Autonomous Equational Categories**

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1. Introduction. A well known phenomenon in the theory of modules is that of "using up operators." A special instance of this consists in the fact that the group $\text{Hom}_\Lambda(A, B)$ of Λ -module homomorphisms between two Λ -modules A and B is in general not a Λ -submodule of the A -fold iterated product B^A unless Λ is commutative. Precisely, unless the action of Λ on B is commutative, there is a Λ -module A , a Λ -homomorphism $f : A \rightarrow B$, and an element $\lambda \in \Lambda$ for which λf is not a Λ -homomorphism. (e.g., take $A = \Lambda$. If $\lambda\mu b \neq \mu\lambda b$, with $\lambda, \mu \in \Lambda$ and $b \in B$, define $f : \Lambda \rightarrow B$ by $f(x) = xb$. Then $(\lambda f)(\mu 1) = \lambda\mu b$, but $\mu(\lambda f)(1) = \mu\lambda b$.) Hence, unless Λ is commutative, there are Λ -modules A and B for which $\text{Hom}_\Lambda(A, B)$ is not a Λ -submodule of B^A .

A similar phenomenon occurs among groups: the set of group homomorphisms from the variable group A to the fixed group B fails to be a subgroup of B^A , for some group A , unless B is abelian.

This paper investigates such phenomena in more detail, in the general setting of equationally definable classes of abstract algebras, using elementary ideas from functor theory. The next section contains the necessary definitions. The main theorem, a characterization of autonomous equational categories, appears in §3. Its proof is postponed until §5, after a short digression, in §4, on tensor products. Some consequences of the main theorem and its proof are presented in §6.

2. Definitions. The category of sets will be denoted \mathbf{S} . In what follows, we will be dealing with categories \mathbf{K} equipped with a fixed faithful "underlying set" functor $A \rightarrow |A| : \mathbf{K} \rightarrow \mathbf{S}$. The hom functor for such a category \mathbf{K} can be lifted to \mathbf{K} over the underlying set functor if there are

- (i) a functor $\mathbf{Hom} : \mathbf{K}^* \times \mathbf{K} \rightarrow \mathbf{K}$, and
- (ii) an equivalence, natural in both A and B ,

$$\eta : |\mathbf{Hom}(A, B)| \xrightarrow{\cong} \mathbf{K}(A, B).$$

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