

Compactification of Homogeneous Spaces, II

ROBERT HERMANN*

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1. Introduction. We present work aiming towards an explanation of the relation between the differential geometry of homogeneous spaces and the properties of the special functions of mathematical physics. Several such possible relations have been sketched in [9] and [10] and this paper is a continuation of that work, rather than a direct sequel to part I [9].

2. The geometric nature of Fuchsian singularities of ordinary differential equations. The theory of radial parts of invariant differential operators [1] plays a key role in the theory of representations of Lie groups, and in the exploration of the connection of this theory with the theory of the special functions of mathematical physics. Now, as we will show in Section 6, these radial operators are operators in manifolds that have singularities lying on certain submanifolds. These singularities seem often to be of the simplest "Fuchsian" type. (For example, it seems to be a very remarkable fact that the spherical functions on many symmetric spaces are hypergeometric functions; of course, the hypergeometric differential equation is characterized by the property of having only Fuchsian singularities.) As preparation for a general explanation of these facts, we will present in this chapter some simple remarks about the geometric meaning of the classical Fuchsian singularities.

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