

# *Generalized Modifications and Cobounding Manifolds*

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**0. Introduction.** In [12] and [13] Wallace studies the relationship between cobounding manifolds and spherical modifications. In §1 we study a more general kind of modification; namely, the (closed) symmetric difference of two manifolds. If  $M_1$  and  $N$  are two intersecting  $n$ -manifolds such that the symmetric difference is again an  $n$ -manifold  $M_2$  we denote  $M_2$  by  $M_1 \Delta N$  and say that  $M_2$  is obtained by modifying  $M_1$  by  $N$ . In §2, we obtain relationships among the homotopy groups of  $M_1$ ,  $N$ ,  $M_2$ .

In §4 we specialize to the case that  $N$  is an  $n$ -sphere  $S^n$ . If certain restrictions are put on  $M_1 \cap S^n$  then  $M_1 \Delta S^n$  is a spherical modification as defined in [12, 504]. The general results on modifications then yield the important theorem on homotopy groups of  $M_1$  and  $M_2$  (Theorem 11.3). In §13 we give conditions under which a generalized modification can be realized by a finite sequence of spherical modifications and in §14 some results are obtained on the existence of manifolds with modification number 1; that is, manifolds upon which functions exist having exactly three critical points.

In [13, 800–801] Wallace introduces the notions of a basic complex and a semimodification and uses them to derive certain results on the kinds of spherical modifications needed to go from one manifold to another if both manifolds and the manifold they bound contain a basic complex. (Cf. [13] Theorem 10.1) An essential ingredient in the proof of Theorems 9.6, 9.7 and 10.3 of [13] is a certain characteristic map  $\Phi$  defined by Wallace in [13, 786]. In §9 we define another map (Case 1) and exhibit its relationship to the map  $\Phi$  (Case 2). This new map enables us to replace the notion of a basic complex by the notion of standard homology form. Theorem 15.1 gives us that any manifold cobounds a manifold in standard homology form and the proof of the theorem gives an algorithm for putting a given manifold in such a form. The corresponding theorem for basic complexes is proven in [13] in a sequence of steps (Lemma 9.1 through 9.5 and Theorems 9.6 and 9.7) by means of semimodifications. The elimination of the use of semimodifications in our method is essentially due to the presence of Theorem 11.3. Theorem 15.2 gives the theorem corre-