

Expansions in Terms of Generalized Heat Polynomials and of Their Appell Transforms¹

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1. Introduction. In a recent paper [8], P. C. Rosenbloom and D. V. Widder discussed expansions of solutions $u(x, t)$ of the heat equation $u_{xx} = u_t$ in series of polynomial solutions $v_n(x, t)$ and of their Appell transforms $w_n(x, t)$. It is our goal to extend this study by considering the generalized heat equation

$$(1.1) \quad \Delta_x u(x, t) = \frac{\partial}{\partial t} u(x, t),$$

where $\Delta_x f(x) = f''(x) + (2\nu/x)f'(x)$, ν a fixed positive number, and by seeking criteria for representing solutions of (1.1) in either the form

$$(1.2) \quad \sum_{n=0}^{\infty} a_n P_{n,\nu}(x, t)$$

or

$$(1.3) \quad \sum_{n=0}^{\infty} b_n W_{n,\nu}(x, t),$$

where $P_{n,\nu}(x, t)$ is the polynomial solution of (1.1) given explicitly by

$$P_{n,\nu}(x, t) = \sum_{k=0}^n 2^{2k} \binom{n}{k} \frac{\Gamma(\nu + \frac{1}{2} + n)}{\Gamma(\nu + \frac{1}{2} + n - k)} x^{2n-2k} t^k,$$

and $W_{n,\nu}(x, t)$ is its Appell transform. Note that $P_{n,0}(x, t) = v_{2n}(x, t)$, the ordinary heat polynomials of even order defined in [8; p. 222]. Also, $P_{n,0}(x, -1) = H_{2n}(x/2)$, the Hermite polynomials of even order defined in [1; p. 222].

We establish the fact that, in general, the series (1.2) converges in a strip

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