

Degree of Approximation of Spline Interpolation

A. SHARMA & A. MEIR

Communicated by G. BIRKHOFF

1. Introduction. Interpolation procedures by "spline-curves" have been recently investigated by several authors ([1], [2], [3], [4]). The main mathematical interest of spline-curves centres around their properties of interpolation and the minimization of certain norms so that these interpolatory functions are in some sense best approximations to a given function also. Our aim here is to obtain results about degree of approximation by spline curves and thereby improve a result of Ahlberg & Nilson [1]. In §2 we state our main results concerning cubic splines, whose proofs are given in §§3 and 4. In §5 we extend some of our results to piecewise quintic polynomials.

2. Convergence of cubic splines. Let

$$(2.0) \quad 0 = x_0 < x_1 < \cdots < x_m = 1$$

be any subdivision of the interval $[0, 1]$. We will prove the following results.

Theorem 1. Let $f(x) \in C^2(-\infty, +\infty)$ with period 1. Let $\varphi(x) \in C^2(-\infty, +\infty)$ be the cubic spline interpolating to $f(x)$ at the joints (2.0) and having period 1. Then for all x

$$(2.1) \quad |\varphi^{(r)}(x) - f^{(r)}(x)| \leq 5\delta^{2-r} \cdot \max_{|u-v| \leq \delta} |f''(u) - f''(v)|, \quad r = 0, 1, 2,$$

where

$$\delta = \max_i (x_i - x_{i-1}).$$

The estimate (2.1) remains valid if the periodicity of $f(x)$ and $\varphi(x)$ is replaced by the requirement

$$(2.2) \quad \varphi'(0) = f'(0), \quad \varphi'(1) = f'(1).$$

Theorem 2. Let $f(x) \in C^3(-\infty, \infty)$ with period 1. For the joints (2.0), let K be such that

$$(2.3) \quad \max_i (x_i - x_{i-1}) \leq K \min_i (x_i - x_{i-1}).$$