*Limit Theorems Involving Capacities**

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1. Introduction. In this paper $\{X_n\}$ will always be an irreducible, transient, Markov chain with states in a denumerable set Ω (which, when convenient, we may label with the non-negative integers), and with n-th step transition probability matrix $P_n(x, y)$. We shall be concerned with the asymptotic behavior of the quantities $P_x(n < V_B < \infty)$ and $P_x(T_B > n)$, where $B \subset \Omega$ is a finite nonempty set, and V_B , T_B are respectively the time of first and last entrance into the set B. When the chain has an invariant measure $\pi(x)$, we shall also investigate the asymptotic behavior of the quantity

$$E_B(n) = \sum_x \pi(x) P_x(V_B \leqq n).$$

Necessary notation and definitions are introduced in Sec. 2

Our principal results are as follows. In Sec. 3 we show that when the chain has an invariant measure $\pi(x)$, then if $C(B)$ is the capacity of B, we have

$$e_B(n) = E_B(n) - E_B(n-1) \downarrow C(B), \qquad n \to \infty, \tag{1.1}$$

and thus

$$E_B(n) \sim nC(B), \qquad n \to \infty. \tag{1.2}$$

Moreover, $E_B(n) - nC(B)$ is nondecreasing, but the limit may either be finite or infinite. The behavior of the above quantity is closely related to that of the quantity

$$\sum_{j=0}^{n} P_x(j < V_B < \infty).$$

Our first task is to establish the conditions for finiteness. In Theorem 3.2 and Corollary 3.1 we show that for each finite nonempty set B and arbitrary state x, if for some state, say 0, we have

$$\sum_{n=0}^{\infty} \sum_{j=n+1}^{\infty} P_j(0, 0) = \sum_{n=0}^{\infty} R_n(0, 0) < \infty, \tag{1.3}$$

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