

# *Parallel Distributions and Contact Transformations*

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The study of contact transformations using the methods of the tensor calculus was initiated by Eisenhart and Knebelman [6]\* who used their "contact frames" to decompose a vector in  $2n$  dimensional space into components in complementary  $n$ -dimensional spaces at any point. Their work was extended by Doyle [5] who made use of non-holonomic subspaces (since called *distributions*) with projection tensors in order to set up what might be called a contact tensor calculus. For his purposes he had to make the assumption that certain tensors vanish, which is equivalent to assuming (in more modern terminology) that the distributions are flat (*i.e.* parallel planes). It was pointed out by the present author in 1954 [3] that Doyle's work could be carried out without making the assumption of flatness. The reason for this has become apparent as an implication of Walker's work on parallel distributions [11], where it is proved that given any such complementary distributions, a global connection can always be found with respect to which these distributions are parallel.

In this paper we re-examine the question of an invariant theory of contact transformations from the point of view of parallel distributions, using a connection which is determined by the almost product structure. We start with a  $C^\infty$  manifold  $V$  of dimension  $N$  on which are defined two complementary  $C^\infty$  distributions  $B$  and  $C$  of dimensions  $n$  and  $N - n$  respectively. Walker has proved [*loc. cit.*] that there exist globally (i) a metric with respect to which  $B$  and  $C$  are mutually orthogonal (ii) a connection with respect to which the metric is constant and the  $B$  and  $C$  are parallel. Walker gives one solution for the connection and points out that other solutions may be obtained by imposing further geometrical conditions. In the first section we prove that if, with respect to a *symmetric* connection, we demand that (i) the distributions are path-parallel (ii) the distributions are parallel relative to each other, then the connection is expressed simply in terms of the metric tensor and the so-called Nijenhuis tensor associated with the distributions.

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\* These numbers refer to the list of papers at the end.