

*Linear Theory of Micropolar Elasticity**

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Abstract. Equations of motion, constitutive equations and boundary conditions are presented for a special class of micro-elastic materials called Micropolar Solids. These solids respond to micro-rotational motions and spin inertia and can support couple stress and distributed body couples. The couple stress theory is shown to emanate as a special case of the present theory when the motion is constrained so that micro- and macro-rotations coincide. Several energy and uniqueness theorems are given.

1. Introduction. The theory of micromorphic materials introduced by Eringen and his co-workers [1] to [5] deals with a class of substances which exhibits certain microscopic effects arising from the local structure and micro-motions of the media. These materials can support stress moments and body moments and are influenced by the spin inertia. The general theory is however very complicated and even in the case of constitutively linear elastic solids [2] the differential equations are not easily amenable to solution.

To simplify the theory Eringen and Suhubi [2] introduced a theory of couple stress. In view of the fact that in the abundant recent literature the terminology of couple stress is used in a different context we thought it may save the reader from confusion if we name our theory "Micropolar elasticity". In contrast to the couple stress theory, in micropolar elasticity all components of the asymmetric stress tensor are determinate and the motion of the media is fully described when the deformation and micro-rotation vectors are known. The concept of micro-rotation and the corresponding field equations are, of course, totally absent in the couple stress theory.

Physically, solids that are composed of dumbbell molecules may be adequately represented by the model of the micropolar elasticity. Fibrous materials and some granular and porous solids may also fall into the domain of this theory.

In Art. 2 we recapitulate the linear theory of microelasticity given in [2] briefly in slightly different form. Art. 3 is devoted to the derivations of the basic equations and boundary conditions of micropolar elasticity. In Art. 4 it is shown that the couple stress theory results as a special case of the micropolar

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