

On the Spectral Determination of the Green's Operator

ROBERT CARROLL*

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1. Introduction. The use of spectral methods has been a valuable tool for the construction of Green's operators (definition below) in the study of certain singular and degenerate Cauchy problems (see [10; 11; 12; 15]). For such problems the operators in question behave irregularly in certain respects and the spectral formulation enables one to concentrate upon the difficulties due to the singularity or degeneracy and to exhibit a framework where continuity, differentiability, *etc.* can be studied easily. We therefore have developed this point of view further in the present paper and in fact a version of theorem 1 below is basic in [12]. The general spectral methods of this paper are seen to exploit a formalism related to the Fourier transformation in say $L^2(\mathbf{R}^n)$ for differential operators with constant coefficients (*cf.* [3; 4]) but involving much more technical apparatus. The hypothesis of commuting families of operators in our case has certain obvious limitations in the case of differential problems on bounded domains Ω with variable coefficients. However for constant coefficients at least there is a relative abundance of admissible differential problems with boundary conditions for such domains (consider for example problems based on a family \tilde{L}_k where $L_k = -\partial^2/\partial x^2 + c_k$ in say $L^2(I_k)$, I_k an interval, $f \in D(L_k)$ requiring f be zero at the endpoints of I_k , $\tilde{L}_k = L_k^{-1}$, and \tilde{L}_k is determined by \tilde{L}_k in $\hat{\otimes} L^2(I_k)$ for the natural Hilbert completion $\hat{\otimes}$ given in [13]). Further developments of these spectral ideas appear in [13; 17; 18].

We will thus consider certain differential problems of the form

$$(1.1) \quad Pu = \sum_{i=0}^m A_i u^{(i)}(t) = f(t); \quad u^{(i)}(\tau) = 0 \quad (i = 0, \dots, m-1)$$

where the A_i are (unbounded) closed densely defined linear operators in a separable Hilbert space H . Various frameworks are possible of course but we will deal with "strong" solutions primarily; thus it is supposed that $t \rightarrow f(t) \in \mathcal{E}^0(H)$ and we try to find solutions $t \rightarrow u(t) \in \mathcal{E}^m(H)$ with $t \rightarrow A_i u^{(i)}(t) \in$

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