

Integration on Real Analytic Varieties I. The Volume Element

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1. Introduction. Suppose V is a complex analytic subvariety of an open subset U in C^n , say of pure dimension k . It has been shown by P. Lelong in [5] that the integral

$$(1.1) \quad \int_V \alpha = \int_{V_r} \alpha,$$

V_r , being the set of regular points of V , exists for any continuous $2k$ -form α on U with compact support.

Suppose, furthermore, that we are given a hermitian metric on U . Let dv be the volume element on V_r associated with the hermitian metric induced on V_r . As indicated in the last corollary of [5], it follows immediately from the above theorem that

$$(1.2) \quad \int_V dv = \int_{V_r} dv$$

converges. In fact, if $\beta = \sum h_{i\bar{j}} dz_i \wedge d\bar{z}_j$ is the two-form associated with the hermitian metric on U , then

$$(1.3) \quad dv = \frac{1}{k!} \left(\frac{i}{2} \beta \right)^k \Big|_{V_r}.$$

Let now V be a real analytic subvariety of an open subset U of R^n and define V_r to be the set of points on V at which V is a manifold of dimension k . Suppose furthermore that V_r can be oriented. We prove that (1.1) exists for a continuous k -form α on U with compact support. This, of course, includes Lelong's result. Our proof seems also to be less involved. However, we should mention that P. Lelong derives with his methods a much deeper result, namely, a special case of Stokes' theorem which we will prove in full generality in a subsequent paper.

If there is given a Riemannian metric on U and if dv denotes the volume element associated with the metric induced on V_r , then we prove that (1.2)