

Integration on Real Analytic Varieties II.

Stokes Formula

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1. Introduction. Let V be a subvariety of some open set in R^n and let V_r^k be the set of regular points on V at which V is of dimension k . Let α be a C^0 differential k -form on R^n . It was shown in [3, section 3] that α is integrable on $V_r^k \cap K$ for any compact subset K of V . In the present paper we show that Stokes' formula holds:

$$\int_D d\alpha = \int_{\partial D} \epsilon_D^k \alpha$$

where α is a differential $(k - 1)$ -form of class C^1 , D is a relatively compact open subset of V^k such that ∂D is analytic and ϵ_D^k is a scalar of odd type taking care of the multiplicity of the boundary.

This solves in particular Conjecture 7.2 in [1], and the construction of a boundary kernel function given in [1] is thus valid for complex analytic spaces.

The results of this paper were announced in [2]. Similar results have been announced by M. Herrera in [7].

2. Regular coordinates. Let V_0 be a germ of real analytic subvarieties at 0 in R^n . We consider R^n embedded in C^n as the real subspace with respect to some coordinate system (z_1, \dots, z_n) for C^n . The germ \tilde{V}_0 of complex analytic varieties at 0 in C^n defined by the ideal of germs of holomorphic functions vanishing on V_0 is called the complexification of V_0 . We recall here some basic properties of the pair (V_0, \tilde{V}_0) that can be found in section 8 of H. Cartan [4].

We have $\tilde{V}_0 \cap R^n = V_0$, and by definition

$$\dim V_0 = \dim \tilde{V}_0.$$

If $V_0 = V_0^1 \cup \dots \cup V_0^s$ is the decomposition of V_0 into irreducible branches then $\tilde{V}_0 = \tilde{V}_0^1 \cup \dots \cup \tilde{V}_0^s$ is the decomposition of \tilde{V}_0 into irreducible branches, and conversely.

A coordinate system (x_1, \dots, x_n) for R^n is said to be a regular coordinate system for the germ V_0 at 0 if the complexification (z_1, \dots, z_n) of (x_1, \dots, x_n)