

# *An Isomorphism Invariant for Spin Factors*

D. M. TOPPING

*Communicated by G. C. ROTA*

**1. Introduction.** In a previous study of weakly closed Jordan algebras of self-adjoint operators (*JW-algebras*) [6], a new factor phenomenon was shown to exist. In the theory of von Neumann algebras, a factor which is of finite class and is type I must be an  $n \times n$  matrix algebra over the complex field. The analogous statement for *JW*-factors is no longer true, the exception being an infinite-dimensional *JW*-factor of the aforementioned class and type. Such a factor was called a "spin factor" in [6] because it was generated by an infinite system of anticommutation relations, more commonly referred to in quantum theory as "Pauli spin relations".

We present here an abstract characterization of spin factors which simultaneously clarifies the construction in [6] and yields a very simple isomorphism invariant. The author was aware of this invariant when [6] was written, and indeed it was mentioned in the first draft of [6], but it was not until recently that the value of an abstract approach became evident. The path taken is comparable to the now classical axiomatization of  $C^*$ -algebras by Gel'fand and Naimark. As far as we know, there is no general set of non-spatial axioms for Jordan operator algebras, except in very particular instances. The present paper fills this gap for spin factors.

A modification of the construction for spin factors given in [6] is made in section 4 (Theorem 1) and shows that the weakly closed linear span of a "spin system" coincides with its uniformly closed linear span. Moreover (Corollary 2), the strong operator and norm topologies agree on a spin factor.

In the last section, we describe the states of a spin factor in complete detail.

The author is indebted to I. Kaplansky for encouraging him to carry out this program, and to N. Jacobson for greatly enhancing his knowledge of the theory of Jordan algebras. Special thanks go to E. Størmer for making available his extensive results in manuscript form, especially [5].

**2. Abstract spin factors.** Let  $A$  be a real Hilbert space (of dimension  $\geq 3$ ) with inner product  $(x | y)$  and norm  $\|x\|_2 = (x | x)^{1/2}$ . Choose and fix a vector  $u \in A$  with  $\|u\|_2 = 1$ , and let  $N = \{u\}^\perp$ . Then  $A = Ru \oplus N$  ( $R$  = the reals). We define a product in  $A$  as follows: