

# *The Initial-Boundary Value Problem for a Non-Linear Hyperbolic Equation in Relativistic Quantum Mechanics\**

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**Abstract.** Considerable attention has been focused on the equation

$$u_{,tt} - \Delta u + u |u|^\gamma = f \quad (\gamma > 0)$$

in the last four years. This paper is concerned with the corresponding initial-boundary value problem. Existence theorems for weak and strict solutions and continuity of weak solutions with respect to the data of the problem are established.

**1. Introduction.** Non-linear hyperbolic equations of the special form

$$(1.1) \quad u_{,tt} - \Delta u + u |u|^\gamma = f \quad (\gamma > 0),$$

have been of considerable interest to mathematicians and physicists in recent years. The case  $\gamma = 2$  in 4-dimensional space-time is important in relativistic quantum mechanics. The function  $u$  represents some field quantity and the inhomogeneous term  $f$  represents sources of energy in the field. Possible interactions of the field with itself are described by the non-linear term. The initial-boundary value problem for equation (1.1) has the following (classical) formulation. It is required to find a function  $u = u(x, t)$  which satisfies (1.1) for  $x \in \Omega$ ,  $t > 0$ , and the initial conditions

$$(1.2) \quad u(x, 0) = u_0(x), \quad x \in \Omega,$$

$$(1.3) \quad u_t(x, 0) = u_1(x), \quad x \in \Omega,$$

and the boundary condition

$$(1.4) \quad u(x, t) = 0, \quad t \geq 0, \quad x \in \partial\Omega.$$

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