

*The Singular, Second Order Oblique Derivative Problem**

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1. Introduction. In recent years there has been considerable interest in generalizing the Schauder estimates proven by Agmon–Douglis–Nirenberg [1] and Browder [5] for the linear elliptic boundary problem

$$(1) \quad \begin{aligned} P(x, D)u &= f \quad \text{in a domain } G, \\ B_t(x, D)u &= \varphi_t, \quad t = 1, 2, \dots, q \quad \text{on } \Gamma = \partial G, \end{aligned}$$

with scalar differentiable polynomials $P(x, D)$ and $B_t(x, D)$. In [1], the Schauder estimates for (1) take the form

$$(2) \quad \|u\|_m \leq C \{ \|P(x, D)u\|_0 + \sum \|B_t(x, D)u\|_{r_t} + \|u\|_0 \}$$

with suitable norms $\|u\|_m$, $\|u\|_{r_t}$ and $\|u\|_0$. Under appropriate smoothness assumptions on problem (1) it was shown in [1] that the estimates (2) hold if and only if the problem satisfies a condition which has been called by various authors regular, elliptic or Shapiro–Lopatinski.

In the case of a second order operator and a single real boundary condition

$$B(x, D)u = \sum_{i=1}^n b_i(x) \frac{\partial u}{\partial x_i}, \quad \text{with } n \geq 3,$$

it is well known that the problem (1), the so called oblique derivative problem, is regular in the above sense if and only if the vector field $(b_1(x), \dots, b_n(x))$ is never tangent to Γ . In this paper we shall be concerned with deriving estimates for second order, elliptic operators with a boundary condition whose associated vector field (b_1, \dots, b_n) is tangent to Γ in a prescribed way; in view of the above remark, such problems are not regular. Problems of this type have been investigated recently by Bitsadze [3], and in a very general context by Hörmander [10], and others. To describe our results more precisely it will be convenient to introduce the following terminology: Let $\partial G = \Gamma$ be an $n - 1$ dimen-

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