

Constitutive Equations for Elastic-Plastic Flow in Cubic Crystals

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1. Introduction. It was shown in a recent article [1] that the elastic-strain relations for cubic crystals can be written in the form

$$(1) \quad \sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} + \alpha e_{km} \nu_a^k \nu_a^m \nu_{ai} \nu_{aj}$$

in a dynamically admissible rectangular coordinate system, where the λ , μ and α are material constants or *elastic moduli* and the ν 's are the components of the three mutually orthogonal unit vectors ν_a which give the orientation of the crystal. Also the δ_{ij} in (1) are Kronecker deltas; the σ_{ij} are the components of the stress tensor and are functions $\sigma_{ij}(x, t)$ of the coordinates x^k and the time t ; and finally the quantities e_{ij} are the components of the usual elastic-strain tensor, *i.e.*

$$(2) \quad e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),$$

where the comma denotes partial differentiation, with respect to the coordinates, of the functions $u_i(x, t)$ which give the components of the elastic displacement.

Now consider a dynamical situation in which plastic as well as elastic deformations occur within the medium. Denote by x_0^i the coordinates of the points P of the unstrained medium, referred to a rectangular system, and by x^i the coordinates of the points Q into which the points P are displaced by the combined elastic and plastic deformation. At any time t the displacement $P \rightarrow Q$ can be represented by equations of the form

$$(3) \quad x^i = A^i(x_0, t).$$

We assume that the functions A^i are continuous and differentiable and that the equations (3) have a unique inverse

$$(4) \quad x_0^i = B^i(x, t),$$

where the B^i are likewise continuous and differentiable functions. Hence the total velocity v of the displaced points Q has components v_i given by

$$(5) \quad v_i = \frac{\partial A^i(x_0, t)}{\partial t} = v_i(x, t),$$