

# *On the Solution to the Plane Problem in the Static Theory of Elasticity for Multiply-Connected Regions with Given Surface Traction*

A. D. FINE & E. N. NILSON

*Communicated by I. N. SNEDDON*

**1. Introduction.** The complete analysis for the plane static problem in classical elasticity was first presented more than thirty years ago by N. I. Muskhelishvili [1]. Since then, many solutions have been obtained via this approach, but in almost all cases, these have involved either simply-connected, doubly-connected, or special multiply-connected regions. A method of solution for the general multiply-connected region was initially suggested by Sherman [2], but until now very little work has been done on this method. In this paper, we generalize Sherman's approach and show that the set of functional forms which he uses represents only a special case of the more general set of functional forms that we allow as being admissible.

**2. Equations for multiply-connected regions.** We consider a finite multiply-connected plane region  $S$ , Figure 1, and denote by  $z = x + iy$  the complex representation of the point  $(x, y)$  in the rectangular Cartesian coordinate system of the plane, with  $i = \sqrt{-1}$ . The displacements in the  $x$ -coordinate and in the  $y$ -coordinate directions are denoted by  $u(x, y)$  and  $v(x, y)$  respectively. The quantities  $\sigma_x(x, y)$ ,  $\sigma_y(x, y)$ , and  $\tau_{xy}(x, y)$  are used to represent, respectively, the normal stress in the  $x$  direction, the normal stress in the  $y$  direction, and the tangential (shear) stress associated with these two directions. If we use the complex potential representation of Muskhelishvili [1], we obtain the following relations:

$$(1) \quad 2\mu\{u(x, y) + iv(x, y)\} = \frac{\lambda + 3\mu}{\lambda + \mu} \Phi(z) - z\overline{\Phi'(z)} - \overline{\Psi(z)},$$

$$(2) \quad \sigma_x(x, y) + \sigma_y(x, y) = 2\{\Phi'(z) + \overline{\Phi'(z)}\},$$

$$(3) \quad \sigma_y(x, y) - \sigma_x(x, y) - i2\tau_{xy}(x, y) = 2\{z\Phi''(z) + \overline{\Psi'(z)}\},$$