

# *A Remark on the Singular Cauchy Problem, for all Values of the Time, for the Euler-Poisson-Darboux Equation*

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**Abstract.** The purpose of the present note is to simplify the treatment of J. B. Diaz and G. S. S. Ludford for obtaining a solution, for all values of the time, to the singular Cauchy problem for the Euler-Poisson-Darboux equation for values of the parameter  $k > m - 1$ . It is verified directly that a certain multiple integral involving the initial value function  $f(x_1, x_2, \dots, x_m)$  provides a solution to the problem. The treatment given here is a modification of an argument due to A. Weinstein. Weinstein's proof requires that  $f$  be of class  $C^3$ , while the present discussion requires only that  $f$  be of class  $C^2$ .

**1. Introduction.** The singular second order partial differential equation

$$(1) \quad \Delta u = \frac{\partial^2 u}{\partial t^2} + \frac{k}{t} \frac{\partial u}{\partial t},$$

where  $k$  is a parameter,

$$\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \dots + \frac{\partial^2}{\partial x_m^2}$$

is the Laplacian, and  $u(x_1, x_2, \dots, x_m, t)$  is a real valued function, is known as the Euler-Poisson-Darboux equation (abbreviated EPD). This singular partial differential equation has occurred, for special values of  $k$  and  $m$ , in many important and classical problems since the time of Euler.

The singular Cauchy problem (one sided, *i.e.*, for  $t > 0$ ) for the EPD equation requires the determination of a real valued function  $u(x_1, \dots, x_m, t)$  which satisfies (1) for  $t > 0$ , and meets the initial conditions

$$(2) \quad u(x_1, \dots, x_m, 0) = f(x_1, \dots, x_m), \quad \frac{\partial u}{\partial t}(x_1, \dots, x_m, 0) = 0,$$

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