

# *An Invariant Formulation of the New Maximum-Minimum Theory of Eigenvalues\**

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**1. Introduction.** This paper is a complement to our previous paper in the same journal [1] and presents our criteria for the inequalities in the maximum-minimum theory in a new form. It could be called an invariant formulation of these criteria and therefore may be of some interest. Using the notation of our previous paper let  $A$  be a compact, negative definite, symmetric operator on a real Hilbert space  $H$  with the scalar product  $(u, v)$ . Denote by  $\lambda_k$ ,  $k = 1, 2, \dots$

$$\lambda_1 \leq \lambda_2 \leq \dots$$

the eigenvalues and by  $u_k$ ,  $(u_i, u_k) = \delta_{ik}$ , the corresponding eigenfunctions (or eigenvectors) of the equation

$$(1) \quad Au = \lambda u.$$

The assumption that  $A$  be negative definite is made only for the convenience of ordering the eigenvalues in an increasing sequence. For the case of infinitely many eigenvalues we have  $\lambda_k \rightarrow 0$ . Let  $2 \leq m \leq n$  where  $m$  is such that

$$(2) \quad \lambda_{m-1} < \lambda_m = \lambda_{m+1} = \dots = \lambda_n \leq \lambda_{n+1} \leq \dots$$

Let  $p_1, p_2, \dots, p_{n-1}$  be any  $n - 1$  elements in  $H$  and denote by  $R(u)$  the Rayleigh quotient  $R(u) = (Au, u)/(u, u)$ . Let us consider

$$(3) \quad \lambda(p_1, p_2, \dots, p_{n-1}) = \min R(u)$$

under the  $n - 1$  orthogonality conditions

$$(4) \quad (u, p_1) = (u, p_2) = \dots = (u, p_{n-1}) = 0.$$

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