

Uniformization of a Quasi-linear Hyperbolic Equation

*Part II. Solution Structure in the Large**

R. E. MEYER

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Abstract. The concepts of regular variables and 'formal' solution (Meyer 1966) are used to study the limit lines of the Generalized Cauchy Problem for a typical system of homogeneous differential equations in two independent and two dependent variables. These lines are of physical interest on account of their relation to bore-formation (in oceanographical applications) and shock-formation (in gas dynamical applications); and of mathematical interest on account of their relation to the breakdown of classical existence theorems.

It is shown that a singular point in the regular plane (corresponding to a limit point) cannot be isolated and a singular curve cannot end in the interior of the domain of the solution. A generalization and new proof are given for Ludford's theorem that any nontrivial solution possesses limit points, if it is continued far enough. These are results in the large concerning domains extending beyond that on which a solution is uniquely determined.

1. Introduction. In the study of the shock-free solutions of quasi-linear hyperbolic equations with two independent variables, attention was attracted early by the connection between limit lines and the breakdown of classical existence of the solutions. Two particular questions arose from this. The first is whether such breakdown is always associated with limit points, and this is answered in the affirmative in Part I (Meyer (1966), Sections and Lemmas of which will be referred to by a prefix I.) for a particular, rather typical class of equations, under conditions eliminating singularities of the equations themselves. The second is whether isolated limit points can occur and whether limit lines can end, questions that obtruded early, since a negative answer was

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