

The Null-Bundle of an Einstein-Riemann Space I: General Theory

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Communicated by V. HLAVATY

1. Introduction. Several years ago, Kerr and Schild [1] obtained a general solution of Einstein's empty space field equations when the metric has the form

$$(1.1) \quad h_{AB} = g_{AB} + \ell_A \ell_B .$$

Here, g_{AB} is the metric of Minkowski space referred to Cartesian coordinates and ℓ_A is a null-vector field: $g^{AB} \ell_A \ell_B = 0$. The method used by Kerr and Schild relied heavily on the fact that g_{AB} was the metric of a flat space, and only the empty space field equations were considered.

This series of papers is devoted to generalizing Kerr and Schild's method of approach to the general case in which the space with metric tensor g_{AB} is not flat and the full Einstein field equations are used. We shall also introduce several alternative methods that naturally arise out of the theory of normed null bases which we shall develop. This will allow us to exhibit a number of new exact solutions.

2. The null-bundle of a hyperbolic-normal metric space. Let g_{AB} denote the metric tensor of a 4-dimensional, hyperbolic-normal metric space E of signature -2 , and let ℓ_A denote the components of a null-vector field in E :

$$(2.1) \quad g^{AB} \ell_A \ell_B = 0 .$$

Definition 2.1. A symmetric tensor field h_{AB} is said to form a *null-argumentation* of g_{AB} if and only if there exists a null-vector field (ℓ_A) on E such that

$$(2.2) \quad h_{AB} = g_{AB} + \ell_A \ell_B .$$

Theorem 2.1. Let the tensor field h^{AB} be defined by

$$(2.3) \quad h^{AB} = g^{AB} - \ell^A \ell^B ,$$

where

$$(2.4) \quad \ell^A = g^{AB} \ell_B .$$