

Determinable Classes of Channels

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In the study of communications theory, it is often convenient to describe a *signal* by a complex valued function $x(t)$ of time t , and to identify the total power of this signal with the integral of $|x(t)|^2$, extended over the time interval of interest. The set of all signals of finite total power then forms a Hilbert space \mathcal{H} , called here the signal space, with norm defined by

$$\|x\|^2 = \int |x(t)|^2 dt.$$

We assume here that the signal $x(t)$ is defined for all t , and vanishes whenever t lies outside the time interval of interest.

It is of practical importance to know what properties of a given signal can be inferred from a finite number of measurements. In particular, can the given signal be distinguished by a finite number of measurements from a host of competing possibilities? A little thought shows that, unless some restriction is placed on the class of competing possibilities, the task of identifying any one of them by a finite number of measurements is hopeless. In fact, for every finite set of measurements there are always pairs of orthogonal signals in \mathcal{H} which yield identical values in each of the measurements in the set.

In the first section of this paper we undertake to describe the kind of limitation which must be imposed *a priori* on a class of signals in order that every signal in the class be distinguishable to within a given mean-square error by a finite number of measurements. Classes of signals which have this property we shall call *determinable classes*.

We first show that every bounded determinable class of signals necessarily has a compact closure in the signal space \mathcal{H} , and conversely, that every class of signals with compact closure in \mathcal{H} is a bounded determinable class. Using this result, we then give various alternative characterizations of bounded determinable classes. In particular, every bounded class of signals whose Fourier coefficients in a given orthonormal expansion die out sufficiently fast is a deter-

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