

# *On the Zeros of Cross-Product Bessel Functions*

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## ABSTRACT

To obtain a useful solution for physical interpretation and eventual numerical evaluation of the wave equation with cylindrical layered geometry, the zeros of certain cross-products of Bessel functions and their derivatives must be determined. In this paper an examination of a set of cross-products related to mixed boundary value problems as analytical functions of the index is carried out. Also the locations of the zeros of these product expressions are compared with the zeros studied by Cochran [1] for different boundary conditions.

**1. Introduction.** In boundary value problems pertaining to cylindrical annuli one encounters the following combinations of functions

$$(1) \quad f_{\nu}(r, kz) = O_{\nu} O_{kz} [J_{\nu}(r) Y'_{\nu}(kz) - J'_{\nu}(kz) Y_{\nu}(r)]$$

with  $O_r = d/d(r)$  or 1, and  $O_{kz} = d/d(kz)$  or 1, where  $J_{\nu}$  and  $Y_{\nu}$  are Bessel functions. These expressions arise regularly in a variety of physical problems like in transmission line theory, acoustics, electric and magnetic theory.

One is usually interested in either the nature of the  $r, kz$ -zeros of the expressions  $f_{\nu}(r, kz)$  for fixed  $\nu$  or that of their  $\nu$ -zeros with  $r$  and  $kz$  given. We are interested in the latter and will derive information about these zeros without the use of numerical computation.

There are then two combinations of operators related to different boundary value problems in two dimensions. For the case of the outer (or inner) boundary condition being Dirichlet's and the inner (or outer) boundary condition being Neumann's we find the following results. When  $z$  and  $r$  are real and  $k > 0$  the appropriate expression

$$J_{\nu}(r) Y'_{\nu}(kz) - J'_{\nu}(kz) Y_{\nu}(r)$$